Motivating Engineering Students to Math Classes: practical experience teaching Ordinary Differential Equations

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***Abstract*—Teaching mathematics for engineering students is very important. New undergraduate students are aware that they will need mathematical concepts in professional activities. It is still usual to observe not only the difficulty of students in mathematical disciplines, but also the lack of motivation for those disciplines. Motivate and keep motivated the engineering students to mathematical disciplines are a challenge for the teacher. In the literature, it is possible to observe the use of modeling tools of mathematical problems, some of them able to simulate models. Those tools have been used to try to motivate students, but with a technological bias that may introduce difficulties. In this paper we propose an approach to teaching-learning process of mathematical concepts to engineering students with a motivational factor: a practical problem to be modeled in which students must apply concepts seen in class, in particular using the Ordinary Differential Equations (ODE). Each student can choose a problem of his/her interest, which increases motivation. The problem chosen must be explored to create mathematical models involving multiple concepts, which allows the comparison of results. Students must create models using existing computational tools, preferably of free use, supported by instructor. So, the students are allowed to apply the concepts acquired to develop projects with transversal themes aiming to verify the capability and the use of these tools in learning mathematical concepts. Also, this paper presents an application of the approach proposed in Numerical Methods for Differential Equations course. We observed that the use of computer tool to model a real problem chosen by students has improved the interest and, consequently, facilitated the teaching-learning process. Also, it may develop in students a new vision on how they can build their knowledge. We also discuss ways in which the computational resources and everyday problems can help in math classes and arouse creativity, motivation and mathematical logical reasoning in engineering students.**

Keywords*—*Motivation; Engineering Students; Ordinary Differential Equations; Computational Tools.

I. Introduction

The Differential Equations have fundamental relevance to applied mathematics area [1], [2]. The study of Differential Equations provides an important mathematical tool to solve different problems from several knowledge area [3], [4]. It is an usual approach to present fundamental concepts on the subject and, then, to show possible applications.

Numerical Methods for Differential Equations is a discipline that focus on numerical methods for solving those equations. Most of the focus on teaching numerical methods is based on theoretical approach. The practical part comes second, and in many cases not even addressed. Thus, students do not experience practicing. To tackle this problem, it is required to students the use of computational tools for understanding the problem.

However, such traditional approach have presented problems on motivating students to mathematical subjects. Our experience on teaching Calculus for engineering students and research in mathematics education area reveal difficulties on teaching-learning process of Ordinary Differential Equations study, both on the use of techniques for solving those equations, as production of meaning and understanding of concepts. The difficulties are particularly evident on studying contextualized problems involving physics, chemistry and engineering. Students dominate the solving techniques, but have difficulty on identifying how to apply the Ordinary Differential Equations to solve problems.

Problem-Based Learning (PBL) is a problem-centered teaching method with exciting potential in engineering education for motivating and enhancing student learning [5]. Implementation of PBL in engineering education has the potential to bridge the gap between theory and practice [6], [7]. However, it is usual to apply PBL by choosing a specific and, then, use the chosen problem as case study for the entire course (even multiples courses for linked-class problem) [5]–[9].

We consider important to students not only the opportunity to apply theoretical information, but also the opportunity to handle situations as similar as possible to real problems. In this paper we present an approach to teaching-learning process of mathematical concepts to engineering students with a motivational factor: a practical problem to be modeled in which students must apply theoretical concepts seen in class, in particular using the Ordinary Differential Equations. But instead of choosing a problem to be dealt with, as has usually been done in the employment of PBL, we ask students to choose problems of their interest.

In this paper we present our approach and discuss the pros and cons observed. The remainder of this paper is organized as follow. In Section II is presented the context required and

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the processes to software development. Section III present an hybrid approach (using characteristics of Problem-Based Learning). Section IV presents examples of real problems and the results obtained by students. Lessons learned, a discussions about the approach application and possible pitfalls are presented in Section V. Related works are briefly presented in Section VI. Finally, Section VII presents the final remarks and further works.

II. The Context: Differential Equations

Differential equations have numerous practical applications in Engineering, Medicine, Chemistry, Biology and other areas of knowledge. They are used to construct mathematical models of physical phenomena, and the strictly mathematical method of solving the model is called analytical method.

It is reasonable to say that the analytical solution is desirable because it allows qualitatively and quantitatively analyzing the phenomenon quite easily. But in many cases, such solution might not exist or be very difficult to be found, or may be too complex for an explicit determination of its variables.

The issue highlights the role of mathematics in the study of scientific problems and engineering. In such problems, one may be interested in a specific answer or one may be interested in investigating the nature of the relationship among the variables involved in the problem.

In this context, the mathematical modeling appears as an art, seeking to formulate and solve real problems using mathematics concepts, in a continuous process of knowledge construction. This interaction (mathematical and real problem) involves a number of procedures that can be grouped, according to [10], into three steps:

1) Interaction: recognition and the theoretical research on problem situation to be resolved.
2) Mathematization: problem formulation through hypotheses and problem resolution based on a model.
3) Mathematical model: analysis and interpretation of the solution, consisting in model validation and evaluation.

The mathematical model is composed by a set of equations. The solution of such equations can be obtained by analytical and numerical methods. And it is usual do not obtain analytical solutions. In those cases, it is required the use of numerical methods that provide approximate solutions to the problem, and it is essential the definition of computational models that allow the simulation of physical processes involved, ensuring a effective result.

The computational advances in recent years were decisive for highlighting numerical simulation in the study of those problems, improving more and more numerical methods. The basic idea of those methods is the discretization process, which reduces continuing problem, with an infinite number of variables, in a discrete problem, with a finite number of variables, which can be resolved computationally.

Thus, the discipline of Numerical Methods for Differential Equations is important for students, since it focus on resolution methods of those differential equations. In particular, it is very important for engineering students. There are several types of numerical methods used to find the solution of an ordinary differential equation, but during the course was focused on simple step methods such as: Explicit Euler, Implicit Euler, Trapezium method, Second Order Taylor method, Runge-Kutta methods (1st, 2nd, 3rd, and 4th order), and Predictor-corrector method. Other methods could be addressed, but we chose them because they are usual on engineering problems.

It is still usual to observe not only the difficulty of students in mathematical disciplines, but also the lack of motivation for those disciplines. Motivate and keep motivated the engineering students to mathematical disciplines are a challenge for the teacher. In the literature, it is possible to observe the use of modeling tools of mathematical problems, some of them able to simulate models. Those tools have been used to try to motivate students, but with a technological bias that may introduce problems.

III. Teaching-learning proposed approach

In this paper we propose an approach the teaching-learning process of mathematical concepts to engineering students with a motivational factor: a practical problem to be modeled in which students must apply concepts seen in class, in particular using the Ordinary Differential Equations (ODE).

First, it is important to note that only one course was conducted using this approach. So, the defined educational outcomes are related specifically to the ordinary differential equation course. In order to motivate the students, teachers must encourage student to find their real problems, not offer a list to them.

The approach consists of the following steps: 1) during the initial class the teacher presents characterizes of problems that can be handled using ordinary differential equations (it is important to avoid examples in order to not induce students on their choices); 2) the students must choose problems, even with no knowledge about ordinary differential equations and present an superficial description; 3) while the teacher presents the fundamentals of ordinary differential equation, students must study their problem in detail, and will be able to formalize the problem. As a mentor, we suggest the teacher to encourage students to choose problems with analytical solutions. Each student must present his/her problem, followed by a discussion about the problems – at this point the teacher must point out why a problem is considered unappropriated, if any (step 2). Step 3 is repeated for each ordinary differential equation method discussed.

Table I presents an schedule for the proposed approach. For that table, each class lasts for four hours. Step 1 is the *class 1*, step 2 is the *class 2*. *Self-Directed Learning* is equivalent to step 3, where students must identify knowledge deficiencies and apply new knowledge about methods. At this point, teacher must act as a guide to direct students to focus on methods according to the content of the discipline, in an appropriated order.

IV. Approach Application and Discussion

The concepts of each method are discussed during the classes, but we do not present them here because they are well known and are out of scope.

TABLE I. PLAN COURSE EXAMPLE

Classes	Content
Class 1	Introduction to differential equations (present problems characteristics)
Class 2	Problem definition followed by discussion and Self-Directed Learning
Class 3	Finite difference method for one-dimensional problem
Class 4	Self-Directed Learning
Class 5	Initial value problem
Class 6	Self-Directed Learning
Class 7	Methods based on Taylor series
Class 8	Self-Directed Learning
Class 9	Methods of Runge-Kutta;
Class 10	Self-Directed Learning
Class 11	Methods of predictor-corrector
Class 12	Self-Directed Learning
Class 13	Convergence, consistency and stability
Class 14	Self-Directed Learning
Class 15	Problem and Results presentation (as seminar)

As mentioned, each student must choose a problem of his/her interest (what increases motivation). In addition, each problem must be explored to create mathematical models involving multiple concepts, which allows the comparison of results. When possible, analytical solution were obtained and compared to numerical solution of nine different methods. For that, students would create models using existing computational tools, preferably of free use. Also, the students were required to apply the concepts to develop projects with traversal themes aiming to verify the capability and the use of these tools in learning mathematical concepts.

In the following we present two examples of real problems chosen by students, a brief description of each one, the methods applied and the results obtained by them.

A. Real Problem 1: car crash test against a hydraulic fender

A hydraulic fender is placed in a laboratory to simulate the impact conditions of a car at high speed.

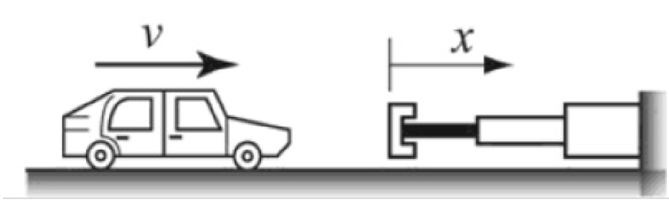

Figure 1. The Problem Model: a car crashing against a hydraulic fender

The fender is designed so that the impact force that the car receives during the collision is a car speed function (v) and displacement (x) of the hydraulic fender axis according to equation

$$F = Kv^2(x+1)^3, \tag{1}$$

where $K = 30(sKg)/m^5$ is a constant.

The deceleration of the car against the fender is given by Newton's Second Law:

$$ma = -Kv^2(x+1)^3. \tag{2}$$

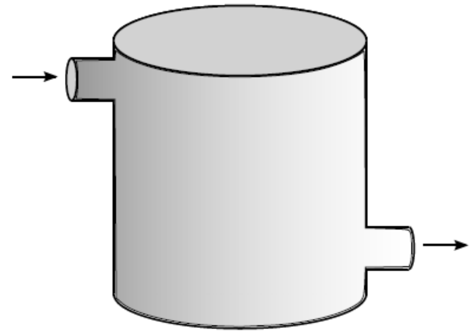

Figure 2. The Problem Model: Tank containing the mixture, and in and out flows

The equation (2) can be solved to express the acceleration in function of x and v:

$$a = \frac{-Kv^2(x+1)^3}{m}. \tag{3}$$

The speed (v) can be calculated as a function of x, replacing equation (3) for the acceleration (a) in the equation:

$$vdv = adx. \tag{4}$$

What results in a first order ordinary differential equation given by

$$\frac{dv}{dx} = \frac{-Kv(x+1)^3}{m}. \tag{5}$$

Consider a car with a weight of $1500Kg$ at a speed of $90km/h$ under test. It is known that the position fender is in the range of 0 and 5 meters. So it is possible determine and plot the car's speed in function of position x for $0 \leq x \leq 5$.

From the first order ODE (5), subject to the initial condition $v = 90km/h$ em $x = 0m$, that must be resolved in the range of $0 \leq x \leq 5$. The analytical solution to this problem is given by

$$v(x) = ve^{\frac{1}{200}1-(x+1)^4}. \tag{6}$$

In this case, all numerical methods have been tested and the results can be seen in Table II.

Among the numerical methods used to determine the car speed against the hydraulic fender, it may be concluded that the fourth order Runge-Kutta method showed higher accuracy because the error is the order of h^5, while others order errors showed are h^4, h^3 and h^2. In addition, the car deceleration was very abrupt when hydraulic fender ran about the range $1.5 \leq x \leq 3.5$.

B. Real Problem 2: mixtures concentration

Suppose a tank containing a mixture of water and salt with an initial volume V_0 (measured in liters) and Q_0 grams of salt. Thus, a saline solution is pumped into the tank at a rate T_e liters per minute having a concentration of C_e grams of salt per liter. Suppose that the well-mixed solution exits at a rate T_s liters per minute.

TABLE II. RESULTS OBTAINED BY THE STUDENT A – INCLUDING ANALYTICAL SOLUTION AND NUMERICAL SOLUTIONS FOR EACH METHOD

x Values (meters)	Exact Solution	Explicit Euler	Implicit Euler	Trapezium	Taylor 2	Runge-Kutta 1	Runge-Kutta 2	Runge-Kutta 3	Runge-Kutta 4	Predictor-Corrector
0	**25**	25	25	25	25	25	25	25	25	25
0,25	**24,82047**	24,875	24,75822	24,81633	24,82844	24,81604	24,82247	24,82051	24,82047	24,81633
0,5	**24,49731**	24,63208	24,34736	24,48853	24,51442	24,48753	24,5014	24,49738	24,49731	24,48854
0,75	**23,97427**	24,21641	23,71195	23,96087	24,00081	23,95836	23,98015	23,97437	23,97427	23,9609
1	**23,19359**	23,56749	22,79995	23,17631	23,22846	23,17102	23,20038	23,1937	23,19359	23,17639
1,25	**22,1034**	22,62479	21,5714	22,08391	22,14369	22,07417	22,10952	22,10354	22,1034	22,08412
1,5	**20,66752**	21,33624	20,00825	20,64845	20,70826	20,63241	20,67075	20,66769	20,66752	20,64892
1,75	**18,87652**	19,66934	18,12367	18,86124	18,91095	18,83754	18,87432	18,87676	18,87652	18,86214
2	**16,758**	17,62404	15,96799	16,74998	16,77859	16,71882	16,7483	16,75834	16,758	16,75146
2,25	**14,38295**	15,24479	13,62875	14,38484	14,38324	14,34918	14,36522	14,3834	14,38296	14,38688
2,5	**11,86465**	12,62817	11,22285	11,87724	11,8415	11,84367	11,84095	11,86509	11,86468	11,87936
2,75	**9,347475**	9,921005	8,881141	9,368978	9,303095	9,347486	9,323038	9,347616	9,347557	9,369997
3	**6,985774**	7,305115	6,728137	7,011905	6,928223	7,0139	6,96829	6,985043	6,98597	7,013132
3,25	**4,916546**	4,967478	4,861975	4,941632	4,85807	4,976351	4,914068	4,914164	4,916949	4,942261
3,5	**3,233494**	3,06082	3,340129	3,25234	3,186732	3,322781	3,25141	3,228696	3,234218	3,252195
3,75	**1,971005**	1,666234	2,174762	1,980722	1,944555	2,081168	2,009473	1,963417	1,972145	1,980924
4	**1,103929**	0,773367	1,338315	1,104783	1,099468	1,221701	1,157298	1,093913	1,105504	1,104723
4,25	**0,562941**	0,290013	0,776503	0,557763	0,575503	0,674184	0,621785	0,551675	0,56485	0,557849
4,5	**0,258868**	0,080184	0,423884	0,251415	0,280006	0,352762	0,313657	0,248008	0,260891	0,25149
4,75	**0,10627**	0,013481	0,217316	0,099536	0,128079	0,177847	0,150633	0,097323	0,108134	0,099581
5	**0,038537**	0,000667	0,104479	0,033915	0,056256	0,088572	0,070493	0,032299	0,040023	0,03394

The amount of salt variation rate in the tank is equal to the entering salt rate in the tank minus the exiting salt rate from the tank. The rate at which salt enters the tank is equal the rate at which the mixture enters, T_e, times the input concentration C_e. The rate at which salt exits the tank is equal the rate at which the mixture exits the tank, T_s, times the salt concentration exiting the tank, C_s.

As the solution is well mixed this concentration is equal to the concentration of salt in the tank, that is,

$$C_s(t) = \frac{Q(t)}{V(t)}. \tag{7}$$

As the volume in the tank, $V(t)$ is equal to the initial volume V_0, added the volume entering at the tank minus the volume exiting the tank, then

$$V(t) = V_0 + T_e t - T_s t = V_0 + (T_e - T_s)t. \tag{8}$$

Thus, the amount of salt in the tank $Q(t)$ is the initial value problem solution:

$$\begin{cases} \dfrac{dQ}{dt} = T_e C_e - T_s \dfrac{Q}{V_0 + (T_e - T_s)t}, \\ Q(0) = Q_0. \end{cases} \tag{9}$$

Considering that a tank has 100 liters of brine containing 30 grams of salt in solution. The water (no salt) enters the tank at a rate of 6 liters per minute and the mixture flowing at a rate of 4 liters per minute, maintaining the concentration uniform by agitating, i.e.,

$$\begin{cases} \dfrac{dQ}{dt} = -4 \dfrac{Q}{100 + 2t}, \\ Q(0) = 30. \end{cases} \tag{10}$$

The analytical solution to this ordinary differential equation is

$$Q(t) = \frac{3 \cdot 10^5}{(100 + 2t)^2}. \tag{11}$$

The range of the variable t is from 0 to 275, divided into 25 sub-ranges, that is, $t \in [0, 275]$ and $n = 25$.

In this case, all numerical methods learned in class have been tested and can be seen in Figures 3 and 4.

C. Other Problems

We do not present here all the real problems chosen by the students. We just selected two of them to show the sort of problems chosen and the results obtained by two students. Other students choose problems in different areas, such as:

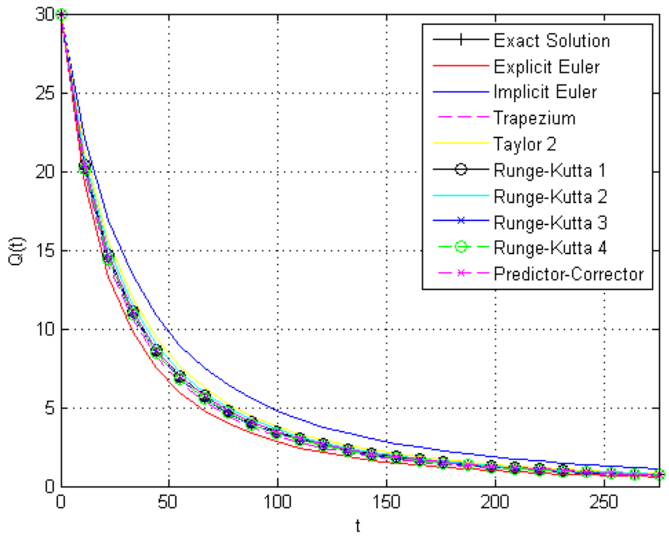

Figure 3. Results obtained by student B: Mixture graphic – Brine Solution.

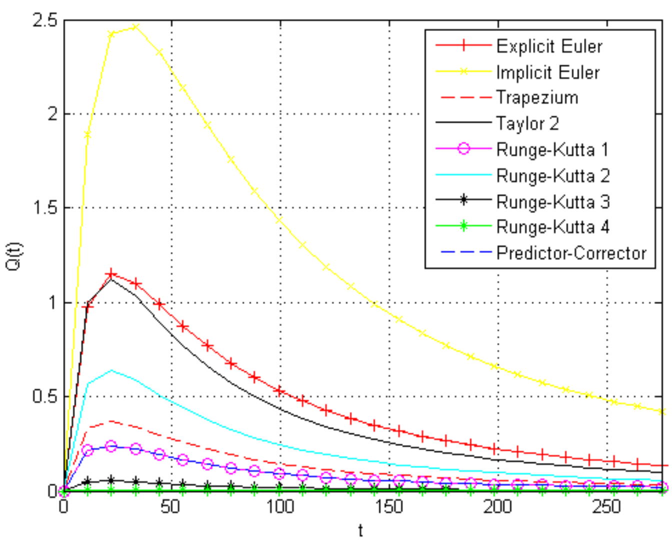

Figure 4. Results obtained by student B: Numerical Methods Errors.

- Pharmacology area (drug absorption and decay drug level in the blood of a patient);
- Physical area (cooling Newton's law);
- Financial area (application of compound interest);
- Demographic area (seasonal variations in populations).

Even not shown here, the results obtained with these problems have been sufficient to observe that students have learned the numerical methods and apply them in the chosen problems.

V. Lessons Learned, Discussions and Pitfalls

In fact, the approach is based on PBL, but it would be considered as a hybrid PBL approach. In a nutshell, PBL learning cycles consist of a problem scenario where students identify facts and generate hypothesis to identify knowledge deficiencies in order to guide self-directed learning, and then, allow students to apply new knowledge [11]. However, PBL requires teachers to choose a well known problem or prepare a list of problems suitable for the subject under study [5], [7]–[9]. And students must use one problem to guide his/her self-directed learning[1].

What is a challenge to teachers on using PBL (to choose adequate problems to be dealt with mathematical subjects) we ask students to choose problems of their interests. Therefore, the first effort required from teachers is motivate students to pick real problems that are meaningful to them. In fact, teachers must act as mentors in order to guide their students on choosing, but with no interference (without influencing their choices). As mentors, teachers must guide students to choose appropriate problems that allow the application of mathematical methods.

Actually, teachers must instigate students to find problems on their real life. However, it is not possible to choose any problem without criteria for several reasons: first, one might pick a problem that would not be possible to handle within course schedule; second, one might pick a problem that would not be possible to solve using only the knowledge in focus; third, the computational knowledge would not be an obstacle; at last, computational resource enough to handle it should be available.

Similarly on implementing PBL, the approach requires resources, planning and organization [12], [13], as usual. However, we recommend the planning task take place after choosing the problems and for each ODE method. A backward planning is enough – the teacher must write down what is expected to learn from doing the project (apply an specific method on the chosen problem). One may observe that in Table I, in which *Self-Directed Learning* classes take place for each method. In fact, we used *Self-Directed Learning* to identify a task that is planned and take place for each method aims at guiding student thinking [11], [14]. Therefore, each planning is specific to the focused method. Also, the computational resource must be available and the student should be able to handle the resources to apply the chosen methods.

It is important to note that our proposed approach was inspired by agile method for software development [15]. Despite its discussion is out of scope, some characteristics are also observed during applying our approach. Similarly to agile methods, students produced iteratively and incrementally results for each ODE method, maximizing opportunities for feedback.

As potential pitfalls we can point out two situations: *Choosing Problems* and *Planning task*. *Choosing Problems* is critical for the approach. The chosen problem must be interesting for student – being related to his/her daily life, facilitates understanding and helps arouse interest. However, the problem can suitable to be handle during the course – one semester. So, too complex problem should be avoided. In addition, one proposed problem drew attention: there was student who proposed as a problem how to cool beer. It is

[1]We are not focusing on PBL, so details on collaborative and geographically distributed teamwork experience are not considered in this paper.

a feasible problem, but obviously may be a signal of social problem – alcoholism. At this point a question arises: leave or refute the problem proposed.

As mentioned, the *Planning task* is specific for each method. We considered that high level instructions and about what is expected to learn from doing the project could be ambiguous. Also, minimally guided instruction is likely to be ineffective [16]. So, it is important to have plannings describing clearly what is expected to learn to each method, how to implement it into computational language and its constraints.

We asked informally to students about advantages and disadvantages of using a problem chosen by them. They consider interesting, since they had the opportunity to develop the ability to study and to apply the knowledge acquired. The background on computational resources was pointed out as difficulty, since the students had to learn the programming language in order to implement the computational solution. But they do not pointed out as a disadvantage.

In addition, the last class – when each student presents his/her results as seminar – is very interesting because each student shares with the class the problem and the results obtained.

Also, during *Self-Direct Learning* classes, we observed that engineering students had initial difficult to implement their solutions (methods). We provide support on algorithms and programming languages in order to allow students to create their solutions. On the other hand, the need to implement their numerical methods was a challenge, motivating the students. The teacher has an important role, giving support so that the difficulties do not become demotivation.

VI. Related work

Cotica and Valencic [12] presented an study about problem-based instruction in mathematics and on affective-motivational aspects, but they focused on nine-year-old students and pointed out that results of the study are useful for the preparation of mathematics curriculum.

Usually, teaching ordinary differential equations uses computational tools [17]–[22]. Computational simulations have been becoming increasingly more prevalent in the education system over the past several decades. Liuxiao [20] discusses how to use problem-based learning strategies to improve the teaching of Numerical Analysis in undergraduate mathematics. The author has systematically studied contemporary education theories, listened to some detailed mathematics classes, and has been exposed to various teaching methods. Many education theories and teaching methods are feasible and applicable to improve teaching. Specially, the ideas of how to stimulate student-centered learning and lifelong learning impressed the author most.

Mrozek [22] shows how advances in computer technology might increase interest in dynamical systems influence by changing the way of teaching ordinary differential equations. He presents inquiry oriented teaching, usage of modeling, visualization and interactive web services. Also, he describes the ways of using MATLAB or public domain software (e.g. Octave) to solve ordinary differential equations. He argues that most students take the differential equations course in order to master techniques to be later applied in solving the real world problems in their profession. Also, he pointed out that students should learn basic concepts that they will remember for the rest of their lives and they should learn how to use the available computer software to model and simulate the real-life applications of differential equations. But he did not point out how to conduct the ODE course.

VII. Final Remarks and Further Work

We believe that the introduction of computers in the educational context is important and it must be exploited to motivate students to practical aspects. Liuxiao [20] pointed out: "the new PBL approaches will give students more motivation for learning and the students will develop independent learning skills which are important for lifelong learning". The use of computer tool to model a real problem chosen by students has empowered the motivation and, consequently, the teaching-learning process.

We took a step forward: we challenge students to bring problems they choose to be dealt with mathematical methods. We consider our approach as a different way to apply PBL. We used such approach during the discipline of Numerical Methods for Differential Equations, and we evaluated the overall approach both by students and the teacher, showing a comparison between a traditional approach purely theoretical and the proposed approach. We notice that students were motivated from the beginning.

Also, the proposed approach may develop in students a new vision on how they can build their knowledge, using computational resources to model his/her chosen problems. We also discuss and analyze the ways in which the computational resources and everyday problems can help in math classes and arouse creativity, motivation and mathematical logical reasoning in students of engineering.

In addition, we observed that students are motivated to share the chosen problem and the results obtained, during the final class. As discussed, we observed that engineering students had difficult implement their solutions (methods), and the support given is very important. The teacher has an important role, giving support so that the difficulties do not become demotivation.

As further work, we intend to use a common problem to a group of students, allowing them to apply different methods to the same problem – so, they would be able work in group, developing their ability for that.

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We intentionally omitted the students authors of chosen problems presented in Section IV.

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